A Comment on “Estimating Dynamic Discrete Choice Models with Hyperbolic Discounting” by Hanming Fang and Yang Wang

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Introduction

Fang and Wang (2015, FW) cited for showing identification of dynamic discrete choice model with hyperbolic discounting by e.g.

▶ Yao, Mela, Chiang, and Chen (2012)
▶ Lee (2013)
▶ Ching, Erdem, and Keane (2013)
▶ Norets and Tang (2014)
▶ Dubé, Hitsch, and Jindal (2014)
▶ Gordon and Sun (2015)
▶ Bajari, Chu, Nekipelov, and Park (2016)
▶ Chan (2017)
▶ Gayle, Golan, and Soytas (2018)

We show that FW's main identification result has no implications for identification (is void) and its proof is incomplete and incorrect.
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We show that FW’s main identification result has no implications for identification (is void) and its proof is incomplete and incorrect.
Overview

**FW's claim:** If there are more (Hotz-Miller and exclusion) restrictions than unknown parameters, then the parameters are “generically identified”: uniquely determined from almost all data (choice and transition probabilities) that the model can generate.

**FW's proof:** Assuming a rank condition holds, the transversality theorem implies that, for almost all data, the model, a system of more equations than unknown parameters, has no solutions “except the true [parameters] that generated the data.”

Note: Argument holds for any smooth system of more equations than unknowns and does not exploit its specific economic structure.

**Our take:** If the rank condition holds, the transversality theorem implies that almost all data are outside the model’s range and has nothing to say about identification from data exceptionally in the model’s range (if the rank condition fails, the transversality theorem has no implications).
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Model

Model is system of $m$ nonlinear equations $F(a, b) = 0$ that relates parameters (discount factors and utilities) $a \in A \subset \mathbb{R}^n$ and data (choice and transition probabilities) $b \in B \subset \mathbb{R}^s$. 

FW does not use (economic) structure of $F(a, b) = 0$. 

A $\rightarrow b$

A $\rightarrow b'$

A $\rightarrow b''$

B $\rightarrow b$

B $\rightarrow a'$

B $\rightarrow a$
Model

Model is system of $m$ nonlinear equations $F(a, b) = 0$ that relates

parameters: discount factors and utilities $a \in A \subset \mathbb{R}^n$
data: choice and transition probabilities $b \in B \subset \mathbb{R}^s$

FW does not use (economic) structure of $F(a, b) = 0$
Model is **identified** if a unique parameter $a \in A$ solves the model $F(a, b) = 0$ for all data $b$. 
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Model is **identified** if a unique parameter $a \in A$ solves the model $F(a, b) = 0$ for all data $b \in B'$.  

Here $B' \subset B$ is the model’s range, the set of data it can generate (formally, set of all $b \in B$ such that $F(a, b) = 0$ for some $a \in A$).
FW focus on generic identification in the data space:

- Model generically identified if unique $a \in A$ solves $F(a, b) = 0$ for all data $b \in B'$ outside set of Lebesgue measure zero in $\mathbb{R}^s$.

To analyze identification, FW apply transversality theorem to $F(a, b) = 0$ with more equations than unknown parameters ($m > n$).
FW assumed that a rank condition for the transversality theorem holds

- Then, the transversality theorem implies that, for all \( b \in B \) outside set of measure zero in \( \mathbb{R}^s \), \( F(a, b) = 0 \) has no solutions \( a \in A \)

FW qualify this with “except the true primitives \([a^*]\) that generated the data” and conclude that the model is generically identified.
FW assumed that a rank condition for the transversality theorem holds

- Then, the transversality theorem implies that, for all \( b \in B \) outside set of measure zero in \( \mathbb{R}^s \), \( F(a, b) = 0 \) has no solutions \( a \in A \)

Correct conclusion from transversality theorem: \( B' \) has measure zero (almost all data reject model), so any claim for \( b \in B' \) outside a set of measure zero is vacuously true
FW assumed that a rank condition for the transversality theorem holds

- Then, the transversality theorem implies that, for all \( b \in B \) outside set of measure zero in \( \mathbb{R}^s \), \( F(a, b) = 0 \) has no solutions \( a \in A \)

Correct conclusion from transversality theorem: \( B' \) has measure zero (almost all data reject model), so unique or multiple \( a \in A \) may solve \( F(a, b) = 0 \) for all \( b \in B' \)
Conclusion

FW’s focus on genericity in data space is nonstandard and renders its results void: Because \( B' \) may have measure zero, a generically identified model can be everywhere or nowhere identified.

FW’s proof is incomplete as it does not verify rank condition for transversality theorem:
- if the rank condition holds, \( B' \) has measure zero, and generic in \( B' \) is void
- if the rank condition does not hold, the transversality theorem does not apply

Either way, the proof has no implications for identification.

The proof is incorrect because it falsely qualifies its conclusion that \( F(a, b) = 0 \) has no solutions \( a \) for \( b \in B \) outside a set of measure zero with “except the true primitives \( [a^*] \) that generated the data.”

FW’s approach does not use the economic structure of \( F(a, b) = 0 \) and can therefore provide no economic intuition for identification.


